

Problem 1

Filament

There is a significant current surge when a filament lamp is first switched on. Propose a theoretical model and investigate it experimentally.

Current Surges

- Filaments increase in temperature when current flows through them
- Resistance increases with temperature, so current decreases with temperature
- Therefore, current decreases over time
- Current starts high when a filament is turned on because its resistance is low when cold

Radiation

- Black bodies radiate with power

$$P_{out} = \sigma AT^4 \quad (\text{Stefan's Law})$$

- They absorb energy from their surroundings in the same manner, giving a resultant power loss

$$P_{Lost} = \sigma A(T^4 - T_0^4)$$

Where T_0 is the temperature of the surroundings

Specific Heat Capacity

- Energy must be provided to raise the temperature of the filament.
- This energy is related to temperature as shown

$$\Delta E = mc\Delta T$$

- This is related to power as shown

$$P = \frac{mc\Delta T}{\Delta t}$$

Producing A Model

- Power in = Power radiated + power to raise temperature of filament

$$IV = \sigma A(T^4 - T_0^4) + \frac{mc\Delta T}{\Delta t}$$

$$\frac{V^2}{R} = \sigma A(T^4 - T_0^4) + \frac{mc\Delta T}{\Delta t}$$

$$\frac{\Delta T}{\Delta t} = \frac{V^2 - R\sigma A(T^4 - T_0^4)}{Rmc}$$

$$\Delta T = \frac{V^2 - R\sigma A(T^4 - T_0^4)}{Rmc} \cdot \Delta t$$

- Temperature change can be calculated and therefore temperature at a specified time can be found

Relating Temperature To Resistance

- From the temperature, the resistance of the filament at a specified time must be found to calculate the current in the filament
- One method for this is to use the assumption that temperature is proportional to resistance

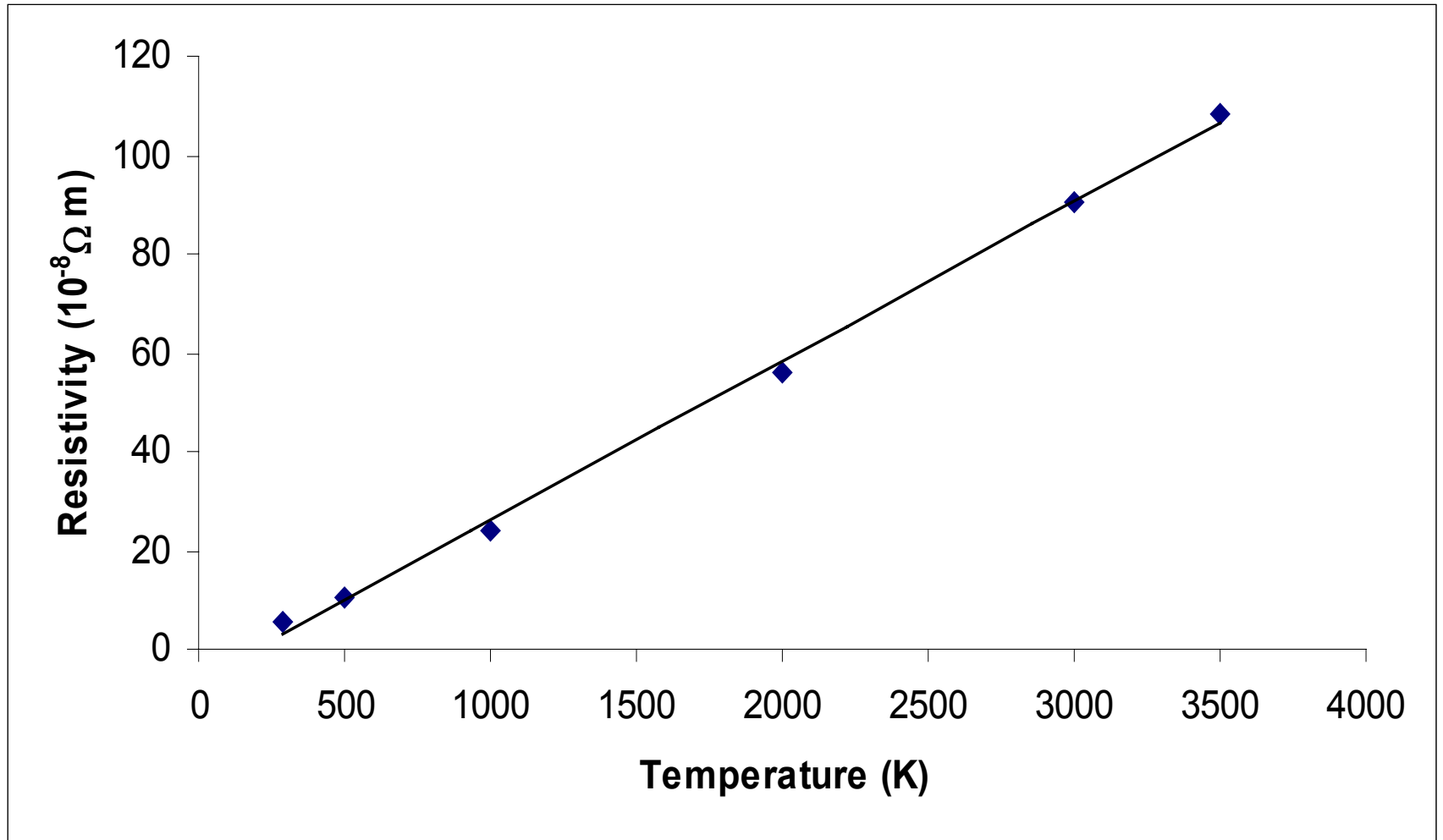
$$R = R_0 (1 + \alpha T)$$

Where R_0 is the resistance at a specified temperature, T is the difference from that temperature and α is the temperature coefficient for the material

Relating Temperature To Resistance

- This formula only holds for some temperature changes and not others
- This can be seen by using values of resistivity at certain temperatures from data tables ($R \propto \rho$)

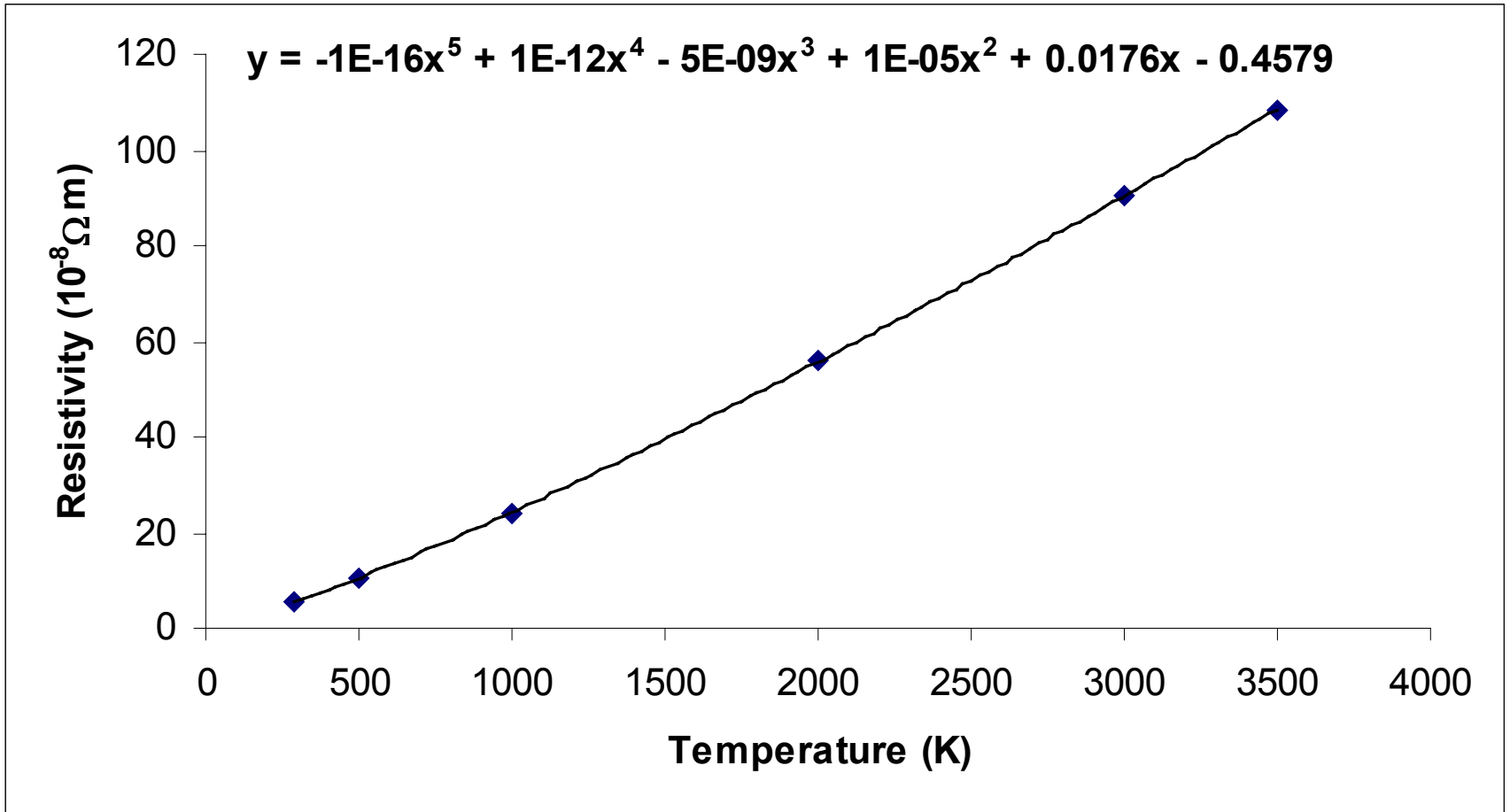
Resistivity Of Tungsten At Various Temperatures



Relating Temperature To Resistivity

- The data shown does not fit a linear trend well enough
- Excel can be used to fit a polynomial to the data and the equation of this can be used to link temperature to resistance in the final model

Resistivity Of Tungsten At Various Temperatures



The Iteration Process

- A start temperature is assigned and a time step length is decided
- The resistance, voltage and temperature are used to predict the temperature change in that time step
- The temperature, resistance and current at the end of the step are calculated
- The process is repeated for the next step

Measuring Parameters

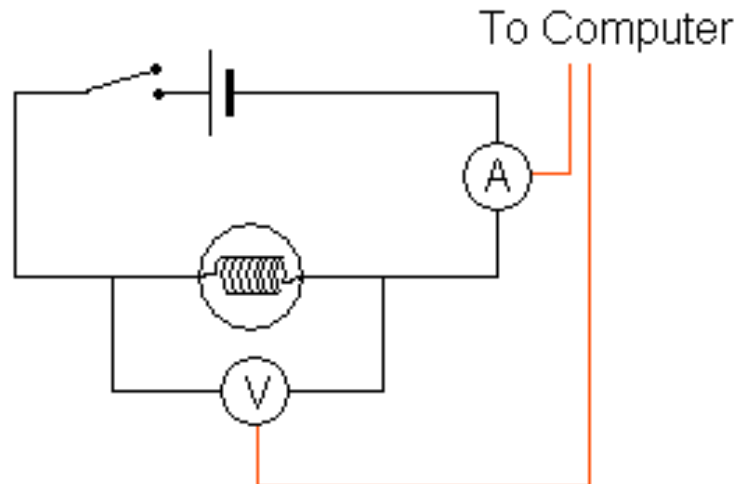
- Bulbs were broken and filaments removed
- Micrometer was used to measure diameter of wire and of coil to $\pm 0.5\mu\text{m}$
- Mass balance measured mass of coil ($\pm 0.05\text{mg}$)

Mass And Length Of Filament

- The length of the filament is difficult to measure
- Number of coils was counted and multiplied by circumference of each coil
- Mass measurement was not accurate enough so volume was calculated and multiplied by density of tungsten

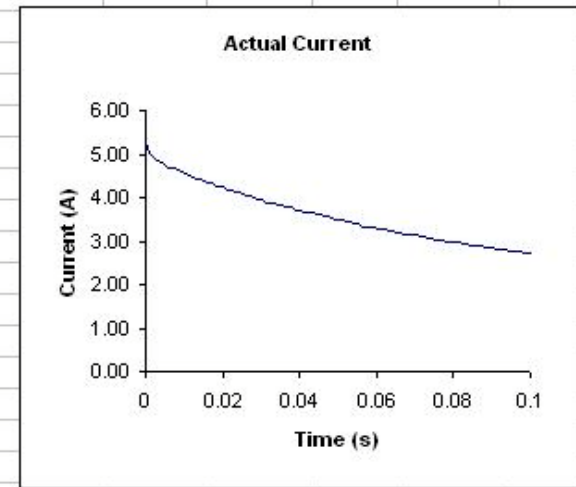
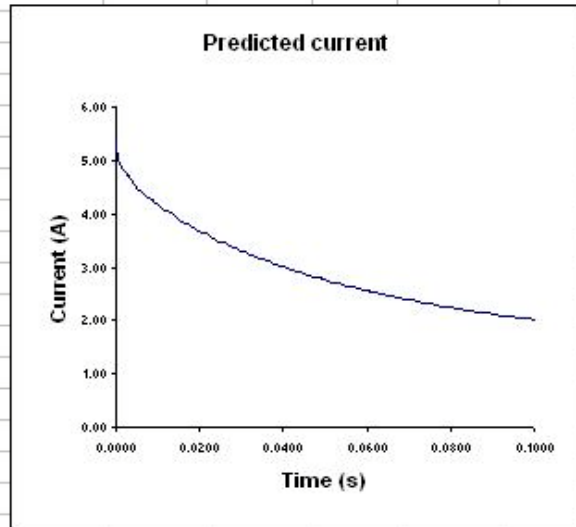
Experimental Results

- Using a 24W, 12V filament bulb, a 6V battery pack, a switch and data-logging voltage and current sensors, current surges can be observed

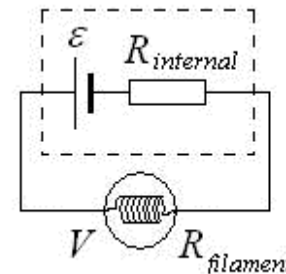


Excel Spreadsheet

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
Radius	Length	Mass	S.H.C.	Sigma	Area	Voltage	Resistivity	Resistance	Time Step	Time	Old Temp	New Temp	Function	Current	Actual Current			
5.1E-05	0.08202	1.29E-05	130	5.6E-08	8.1E-09	2.95	5.46E-08	5.51E-01	0.0005	0.0000	293	297.73105	9.46E+03	5.36	5.27			
						2.87	5.58E-08	5.62E-01		0.0005	297.731	302.11845	8.77E+03	5.11	5.03			
						2.84	5.68E-08	5.72E-01		0.0010	302.1185	306.33538	8.43E+03	4.96	4.94			
						2.83	5.78E-08	5.83E-01		0.0015	306.3354	310.44961	8.23E+03	4.86	4.85			



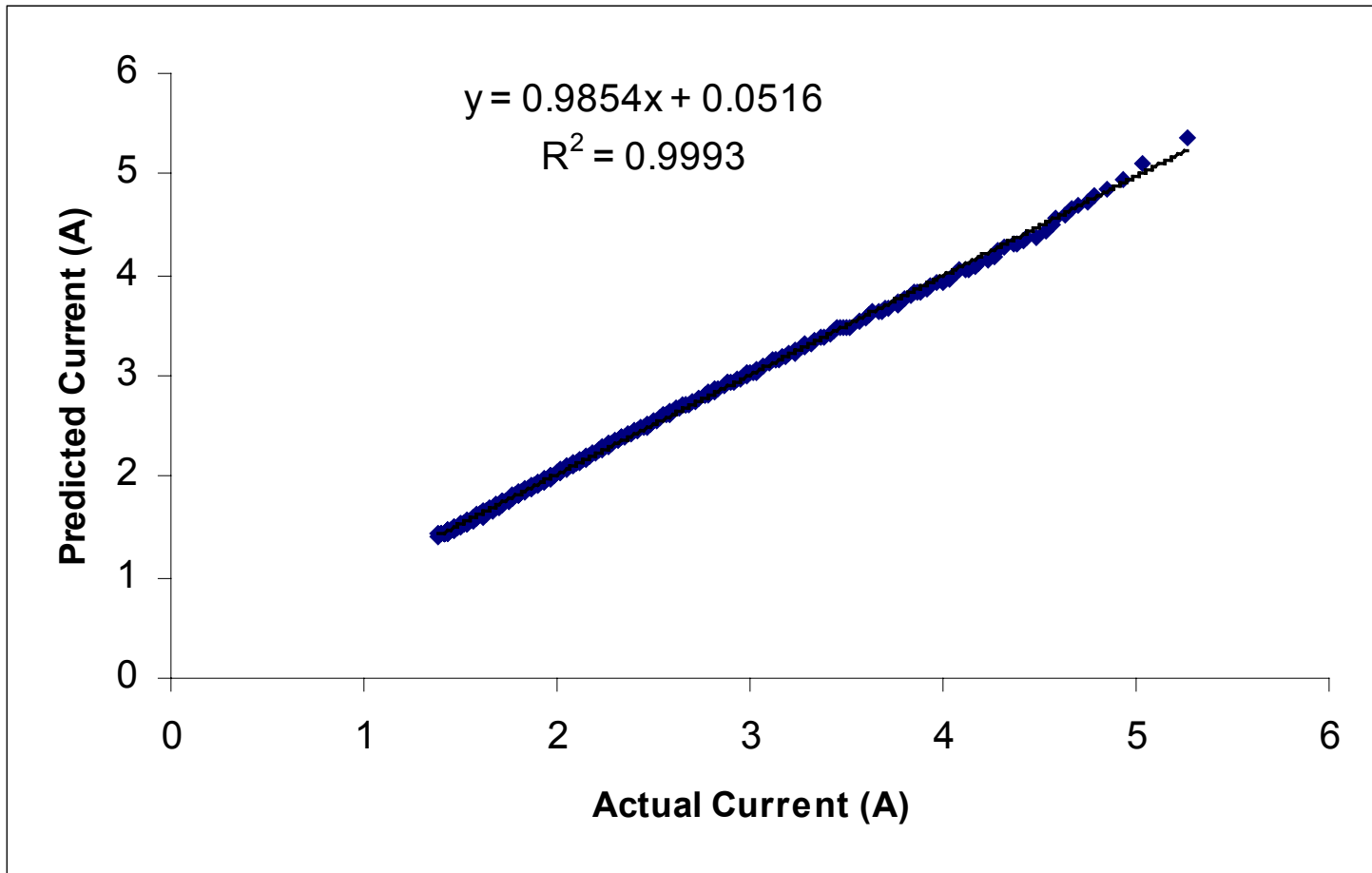
Voltage must be measured as current changes since potential divide exists in the circuit



$$V = \frac{\epsilon R_{filament}}{R_{internal} + R_{filament}}$$

3.01	6.93E																	
3.04	7.02E																	
3.07	7.12E																	
3.09	7.22E																	
3.09	7.31E																	
3.10	7.41E																	
3.11	7.5E																	
3.13	7.59E																	
3.15	7.69E																	
3.17	7.78E																	
3.21	7.88E																	
3.22	7.97E																	
3.23	8.07E																	
3.24	8.16E																	
3.26	8.26E																	
3.27	8.35E-08	8.41E-01								0.0150	411.9302	415.72393	7.59E+03	3.89	3.94			
3.28	8.44E-08	8.51E-01								0.0155	415.7239	419.49778	7.55E+03	3.85	3.92			
3.30	8.54E-08	8.60E-01								0.0160	419.4978	423.27538	7.56E+03	3.84	3.89			
3.33	8.63E-08	8.70E-01								0.0165	423.2754	427.07966	7.61E+03	3.83	3.87			

24W Bulb Using 6V Battery Pack

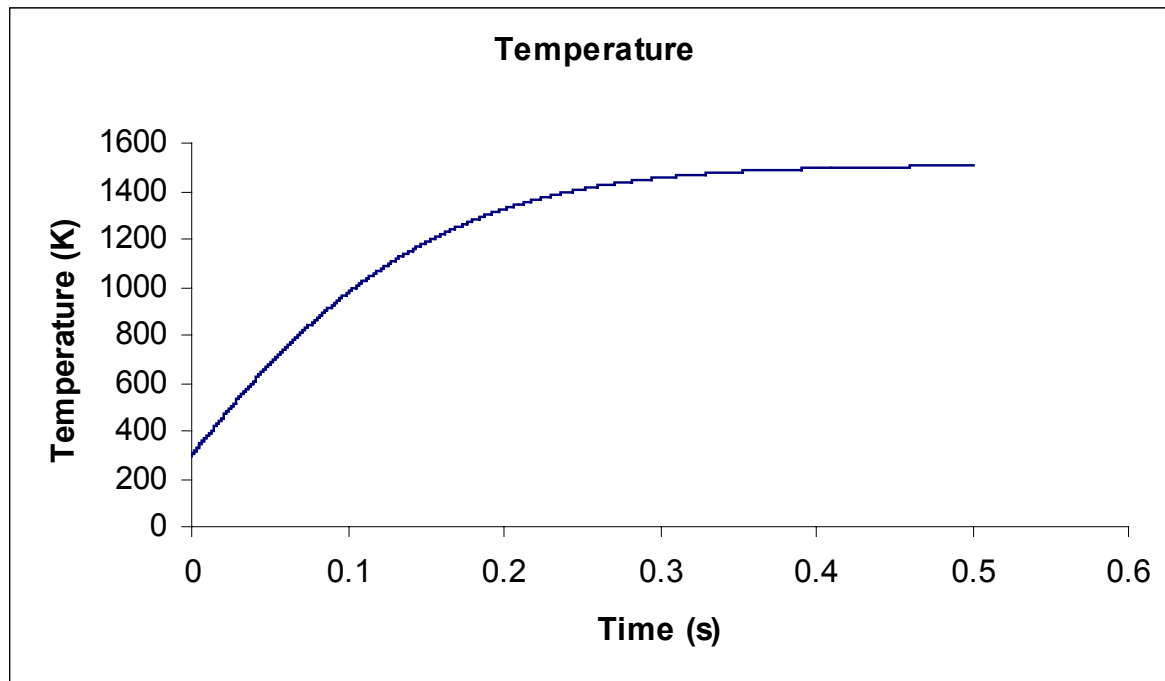


Analysis

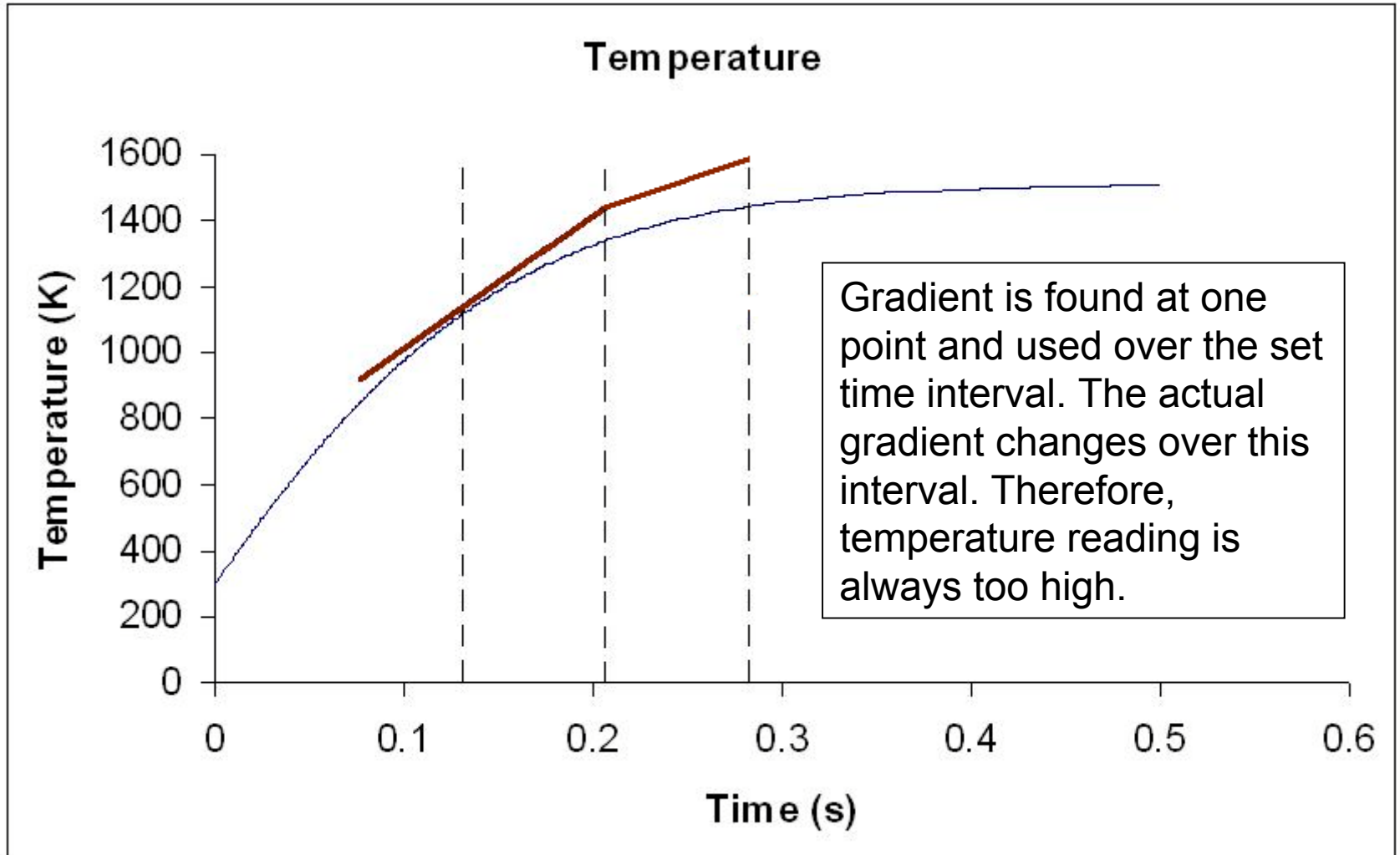
- Model is supported by data (high correlation)
- Intercept should be 0, but is 0.0516
- Gradient is less than 1 (0.9851)

Analysis – Shallow Gradient

- Method of obtaining current at one time relies on previous calculations due to the nature of the iterative process



Analysis – Shallow Gradient



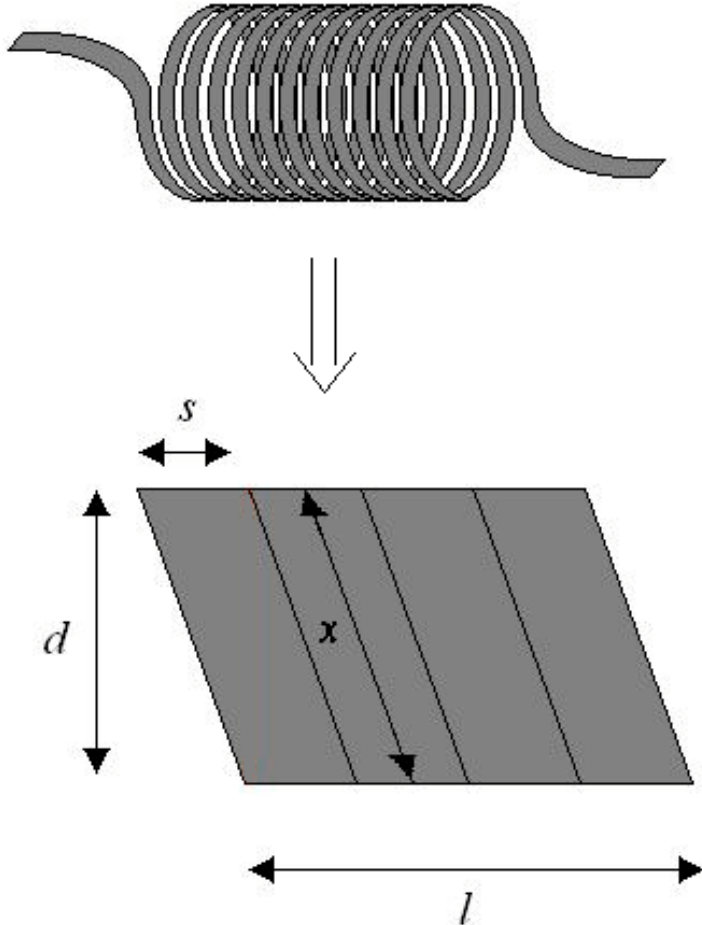
Analysis – Shallow Gradient

- If temperature is lower than calculated value, how should the resistivity and therefore current differ from their calculated values?
- If $\rho \propto T$ (suitable for this analysis), resistivity is also lower than calculated and current is higher than calculated since
$$I \propto \frac{1}{\rho}$$

Analysis – Shallow Gradient

- This could explain the difference between data and calculated values
- This may, however, be a less significant error than other ones, e.g. measurement errors for filament

Error For Coil Length



- Let $n = \text{number of coils}$
- Lower picture, $n = 4$

$$x = \sqrt{d^2 + s^2}$$

$$\text{error} = \sqrt{d^2 + s^2} - d$$

$$\text{total length error} = n \left(\sqrt{d^2 + s^2} - d \right)$$

$$\text{total length error} = 2.053 \times 10^{-4} \text{ m}$$

Measurement Errors

- Ammeter: $\pm 0.005\text{A}$
- Voltmeter: $\pm 0.005\text{V}$
- Filament diameter: $\pm 0.5\mu\text{m}$
- Coil length: $\pm 0.5\mu\text{m}$
- Coil diameter: $\pm 0.5\mu\text{m}$
- Filament length: $\pm 15.5\mu\text{m} + 205.3\mu\text{m}$

Checking Predicted Temperature

- The spreadsheet used calculates the temperature of the filament
- This can be compared to the actual temperature, measured from its emission spectrum

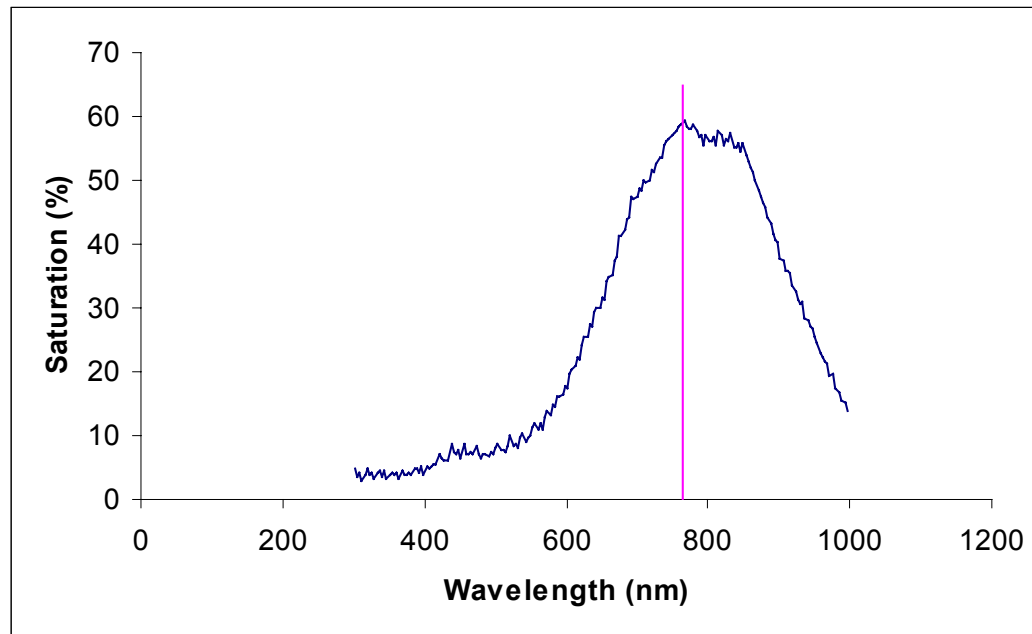
Emission Spectra

- Emission spectra show the relative intensities of different wavelengths of light
- The most prominent wavelength can be used to calculate the temperature of the source of the light from Wien's Law:

$$T = \frac{2.898 \times 10^{-3}}{\lambda_{Peak}}$$

Measuring Temperature

- Use DIVA spectroscope and data analysing software to determine emission spectrum



$$T = \frac{2.898 \times 10^{-3}}{765.66 \times 10^{-9}} = 3785K$$

Checking Predicted Temperature

- Model predicts temperature of 1505K
- This is very different from the measured temperature, however, the DIVA device must be calibrated in order to give a more accurate value of the temperature